VIII Encontro da Pós-Graduação em Matemática da UFBA - EPGMAT

Comportamento assintótico de um passeio aleatório com reforços

Rodrigo Lambert (com M. González-Navarrete and V. Vázquez Guevara)

A general setup

For a sequence of random variables W_1, W_2, W_3, \cdots , we consider

$$S_n = \sum_{i=1}^n W_i \; .$$

For a sequence of random variables W_1, W_2, W_3, \cdots , we consider

$$S_n = \sum_{i=1}^n W_i \; .$$

One can ask about the asymptotics of S_n . That's it, what happens to

 $\lim_{n\to\infty}S_n$

?????

Our model

- 1. At time n = 0 the RW is at the origin; i.e., $S_0 = 0$.
- 2. At time n = 1, we have that $S_1 = X_1$ has the following distribution

$$X_1 = \begin{cases} 1, & \text{with probability } p, \\ -1, & \text{with probability } q, \\ 0, & \text{with probability } r \end{cases}$$

1. For each $n \ge 2$ set

$$X_{n+1} = Y_n \alpha_{n+1} X_{U_n} + (1 - Y_n) \alpha_{n+1},$$
(1)

where; for each $n \ge s$, U_n is a discrete uniform random variable on $\{1, 2, ..., n\}$, Y_n posses the Bernoulli distribution with parameter $\theta \in [0, 1)$ and

Furthermore we set

$$\alpha_{n+1} = \begin{cases} 1, & \text{with probability } p, \\ -1, & \text{with probability } q, \\ 0, & \text{with probability } r \end{cases}$$

with p + q + r = 1. In addition, we assume that α_n , and U_n are independent and that Y_n is independent of the RW's past.

The position of the random walk at time $n \ge 0$ is given by

$$S_{n+1} = S_n + X_{n+1}.$$
 (2)

The position of the random walk at time $n \ge 0$ is given by

$$S_{n+1} = S_n + X_{n+1}.$$
 (2)

What about the asymptotics of S_n ?

$$\alpha = \theta(p-q) ,$$

we will study three cases

$$\alpha = \theta(p-q) ,$$

we will study three cases

• If $\alpha < 1/2$, we are in the diffusive case

$$\alpha = \theta(p-q) \; ,$$

we will study three cases

- If $\alpha < 1/2$, we are in the diffusive case
- If $\alpha = 1/2$, we are in the critical case

$$\alpha = \theta(p-q) \; ,$$

we will study three cases

- If $\alpha < 1/2$, we are in the diffusive case
- If $\alpha = 1/2$, we are in the critical case
- If $\alpha > 1/2$, we are in the superdiffusive case

The diffusive regime

Theorem

If $\alpha < 1/2$, then we have the following almost sure convergence

$$\lim_{n \to \infty} \frac{S_n}{n} = \frac{\omega}{1 - \alpha},\tag{3}$$

where $\omega = (1 - \theta)(p - q)$. to be precise

$$\left(\frac{S_n}{n} - \frac{\omega}{1 - \alpha}\right)^2 = O\left(\frac{\log n}{n}\right), \quad a.s.$$
(4)

The critical regime

Theorem

If $\alpha = 1/2$, then we have the following almost sure convergence

$$\lim_{n \to \infty} \frac{S_n}{n} = 2\omega,$$
(5)

more precisely

$$\left(\frac{S_n}{n} - 2\omega\right)^2 = O\left(\frac{\log n \log \log n}{n}\right), \quad a.s.$$
(6)

The superdiffusive regime

Theorem

If $\alpha > 1/2$, we have the almost sure convergence

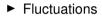
$$\lim_{n \to \infty} n^{1-\alpha} \left(\frac{S_n}{n} - \frac{\omega}{1-\alpha} \right) = L \qquad \text{a.s.}$$
(7)

and L is a non-degenerated random variable such that

$$\mathbb{E}[L] = \frac{\beta(1-\alpha) - \omega}{\Gamma(\alpha+1)(1-\alpha)}$$
(8)

where $\beta = p - q$.

What's next?



- Fluctuations
- ► Higher dimensions (obvious!!!)

- Fluctuations
- ► Higher dimensions (obvious!!!)
- Hypothesis testing (for the process $(X_n)_{n \in \mathbb{N}}$)

- Fluctuations
- ► Higher dimensions (obvious!!!)
- ► Hypothesis testing (for the process $(X_n)_{n \in \mathbb{N}}$)
- Pólya urn model approach

Muito obrigado!!!