VIII Encontro da Pós-Graduação em Matemática da UFBA - EPGMAT

Comportamento assintótico de um passeio aleatório com reforços

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A general setup

For a sequence of random variables $W_1, W_2, W_3, \cdots$, we consider

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One can ask about the asymptotics of S_n. That's it, what happens to

$$\lim_{n \to \infty} S_n$$

?????

Our model

1. At time $n = 0$ the RW is at the origin; i.e., $S_0 = 0$.
2. At time $n = 1$, we have that $S_1 = X_1$ has the following distribution

$$X_1 = \begin{cases} 1, & \text{with probability } p, \\ -1, & \text{with probability } q, \\ 0, & \text{with probability } r \end{cases}$$

1. For each $n \geq 2$ set

$$X_{n+1} = Y_n \alpha_{n+1} X_{U_n} + (1 - Y_n)\, \alpha_{n+1}, \tag{1}$$

where; for each $n \geq s$, U_n is a discrete uniform random variable on $\{1, 2, \ldots, n\}$, Y_n posses the Bernoulli distribution with parameter $\theta \in [0, 1)$ and

Furthermore we set

$$\alpha_{n+1} = \begin{cases} 1, & \text{with probability } p, \\ -1, & \text{with probability } q, \\ 0, & \text{with probability } r \end{cases}$$

with $p + q + r = 1$. In addition, we assume that α_n, and U_n are independent and that Y_n is independent of the RW's past.

The position of the random walk at time $n \geq 0$ is given by

$$S_{n+1} = S_n + X_{n+1}. \tag{2}$$

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What about the asymptotics of S_n?

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we will study three cases

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- If $\alpha < 1/2$, we are in the diffusive case
- If $\alpha = 1/2$, we are in the critical case
- If $\alpha > 1/2$, we are in the superdiffusive case

The diffusive regime

Theorem

If $\alpha < 1/2$, then we have the following almost sure convergence

$$\lim_{n\to\infty} \frac{S_n}{n} = \frac{\omega}{1-\alpha}, \tag{3}$$

where $\omega = (1-\theta)(p-q)$. to be precise

$$\left(\frac{S_n}{n} - \frac{\omega}{1-\alpha}\right)^2 = O\left(\frac{\log n}{n}\right), \quad \text{a.s.} \tag{4}$$

The critical regime

Theorem

If $\alpha = 1/2$, then we have the following almost sure convergence

$$\lim_{n\to} \frac{S_n}{n} = 2\omega, \tag{5}$$

more precisely

$$\left(\frac{S_n}{n} - 2\omega\right)^2 = O\left(\frac{\log n \log\log n}{n}\right), \quad \textit{a.s.} \tag{6}$$

The superdiffusive regime

Theorem

If $\alpha > 1/2$, we have the almost sure convergence

$$\lim_{n\to\infty} n^{1-\alpha}\Big(\frac{S_n}{n} - \frac{\omega}{1-\alpha}\Big) = L \qquad \text{a.s.} \tag{7}$$

and L is a non-degenerated random variable such that

$$\mathbb{E}[L] = \frac{\beta(1-\alpha) - \omega}{\Gamma(\alpha+1)(1-\alpha)} \tag{8}$$

where $\beta = p - q$.

What’s next?

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- Pólya urn model approach

Muito obrigado!!!