

# Gain of regularity for a coupled system of generalized nonlinear Schrödinger equations

Universidad de Tarapacá  
Arica-Chile

Raul Nina Mollisaca

**Work joint with:** Dr. Mauricio Sepúlveda Cortés and Dr. Octavio Vera Villagran  
The author is partially financed by project Fondecyt 1220869.

# The motivation

**Natural phenomenon:** Solids can be classified on the basis of their internal structure: crystalline and amorphous. In certain crystalline materials, such as calcite and quartz, the speed of light does not is the same in all directions (anisotropic material)

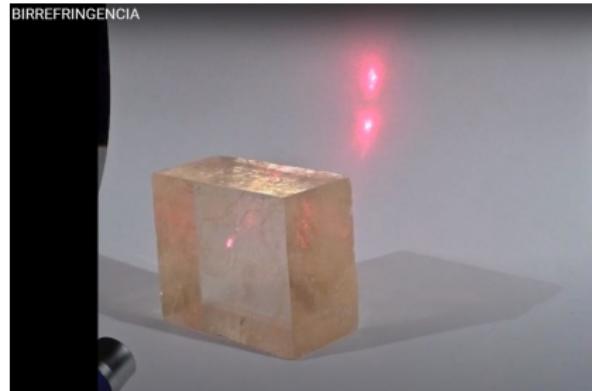


Cuarzo and Calcita.

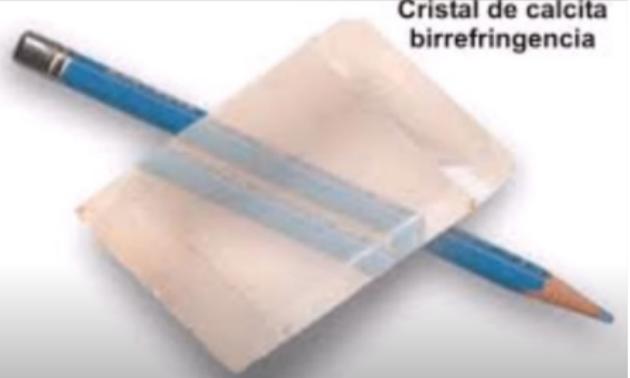
These materials are often called double refractive or birefringent.

# The motivation

When a ray of light passes through an anisotropic medium, it breaks down into two rays, one ordinary and the other extraordinary. [G.P. Agarwal, 1995] and [R.A. Serway, J.W. Jewett Jr, Vol 2, 2014].



Cristal de calcita  
birrefringencia



Crystal of calcita.

# Mathematical model

We have

$$\begin{cases} iu_t + u_{xx} = (|u|^{2p} + \beta|u|^{p-1}|v|^{p+1})u, \\ iv_t + v_{xx} = (|v|^{2p} + \beta|v|^{p-1}|u|^{p+1})v, \end{cases} \quad (1)$$

where  $t, x \in \mathbb{R}$ ,  $p > 1$  is a number odd integer (technical hypothesis),  $u = u(x, t)$  y  $v = v(x, t)$  are complex functions called wave function. The  $\beta$  parameter is a positive real constant and is interpreted as birefringence intensity,  $u_t$  y  $u_{xx}$  are the time and space derivatives of wave functions.

For the correct statement of the problem, the system (1) is complete with initial conditions

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x) \quad \text{en } \mathbb{R}, \quad (2)$$

Se observa que si en (1) se hace  $\beta = 0$ , entonces este se reduce al modelo clásico de Schrödinger.

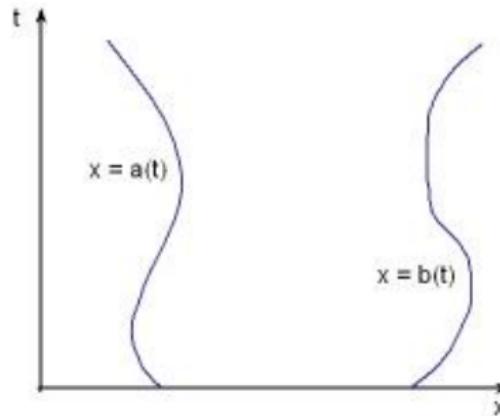
$$\begin{cases} iu_t + u_{xx} = |u|^{2p} u, \quad x, t \in \mathbb{R} \\ u(x, 0) = u_0(x). \end{cases} \quad (3)$$

# The Heat Equation

We consider the region (open and connected):

$$G = \{(x, t) \in \mathbb{R}^2 / 0 < t < T, a(t) < x < b(t)\}$$

where  $a(t), b(t)$  are curves given for  $0 \leq t \leq T$ .



We consider the Heat Equation:

$$u_t = u_{xx}, \quad (x, t) \in G. \quad (4)$$

# The Heat Equation

Problem. Find a function  $u \in C^2(G)$  such that  $u_t = u_{xx}$ .

If  $u$  exists, then  $u$  is called solution of the Heat Equation (4).

**Theorem.-** There is at most one solution  $u(x, t)$  in the region  $G$  of the Dirichlet initial-boundary-value problem (PVIF)

- ①  $u_t = u_{xx} + F(x, t)$ , para todo  $a(t) < x < b(t)$ ,
- ②  $u(a(t), t) = g_1(t)$  y  $u(b(t), t) = g_2(t)$ ,  $0 < t < T$ ,
- ③  $u(x, 0) = u_0(x)$ ,  $a(0) < x < b(0)$ .

# The Heat Equation

**Definition.-** The solution fundamental de (1) is define by the function

$$K(x, t) = \frac{1}{\sqrt{4\pi t}} e^{-\frac{x^2}{4t}}, \quad x \in \mathbb{R}, \quad t > 0.$$

**Lemma.-**

- 1  $K_t(x, t) - K_{xx}(x, t) = 0.$
- 2  $K(x, t) > 0.$
- 3 Si  $t > 0$ ,  $K(x, t) \rightarrow 0$  exponencialmente cuando  $|x| \rightarrow \infty$ , y lo mismo vale para las derivadas de  $K(x, t)$ .
- 4 Para todo  $\delta > 0$ ,  $\lim_{t \rightarrow 0^+} K(x, t) = 0$  uniformemente sobre  $\{x/|x| \geq \delta\}$ .
- 5  $\int_{-\infty}^{\infty} K(x, t) dx = 1.$
- 6  $\lim_{t \rightarrow 0^+} \int_{|x| \geq \delta} K(x, t) dx = 0$  para todo  $\delta > 0$ .

# The Heat Equation

**Theorem.-** Let  $f(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$  be a bounded and continuous function. Define  $u(\cdot, \cdot) : \mathbb{R} \times [0, \infty) \rightarrow \mathbb{R}$  by

$$u(x, t) = \begin{cases} \int_{-\infty}^{\infty} K(x - \xi, t) f(\xi) d\xi, & t > 0, \\ f(x), & t = 0. \end{cases} \quad (5)$$

Then  $u(\cdot, \cdot)$  is bounded and continuous on  $\{(x, t)/t \geq 0\}$ , it is infinitely differentiable in  $\{(x, t)/t > 0\}$ , and it satisfies the initial-value problem

$$\begin{cases} u_t(x, t) - u_{xx}(x, t) = 0, & x \in \mathbb{R}, t > 0, \\ u(x, 0) = f(x). \end{cases} \quad (6)$$

# The Heat Equation

**Remark.-**

$$u(x, t) = (K(., t) * f)(x)$$

Doing calculations, we obtain  $u(., t) = S(t)f(.)$ ,  $t > 0$ , where  $S(t) : X \rightarrow X$ ,  $X$  is the  $\mathbb{R}$ -linear space of continuous and bounded functions, we have  $S(t) = e^{-At}$ ,  $S(0) = I$ ,  $S(t + \tau) = S(t) \circ S(\tau)$ ,  $t, \tau > 0$  and  $A : X \rightarrow X$  define by

$$A(u) = f \Leftrightarrow u, f \in X \text{ and } -u''(x) = f(x),$$

for all  $x \in \mathbb{R}$ .

# Mathematical model

We have the coupled system of generalized nonlinear Schrödinger equations

$$\begin{cases} iu_t + u_{xx} = (|u|^{2p} + \beta|u|^{p-1}|v|^{p+1})u, \\ iv_t + v_{xx} = (|v|^{2p} + \beta|v|^{p-1}|u|^{p+1})v, \\ u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x) \quad \text{en } \mathbb{R}, \end{cases} \quad (7)$$

# Solutions of finite energy

**Lemma 1.-** Let  $u$  and  $v$  be the solutions of (1), then we have

$$(a) \frac{d}{dt}(|u|^2) = 2\operatorname{Im}(u\bar{u}_{xx}) \quad \text{and} \quad (b) \frac{d}{dt}(|v|^2) = 2\operatorname{Im}(v\bar{v}_{xx})$$

**Proof.-** Caso (a). Multiplying (1)<sub>1</sub> by  $\bar{u}$  we have

$$i\bar{u}u_t + \bar{u}u_{xx} = (|u|^{2p} + \beta|u|^{p-1}|v|^{p+1})|u|^2. \quad (8)$$

Applying conjugate in the above equation

$$-iu\bar{u}_t + u\bar{u}_{xx} = (|u|^{2p} + \beta|u|^{p-1}|v|^{p+1})|u|^2. \quad (9)$$

Subtracting (8) with (9) we obtain

$$\begin{aligned} i\frac{d}{dt}(|u|^2) + \bar{u}u_{xx} - u\bar{u}_{xx} &= 0 \iff i\frac{d}{dt}(|u|^2) = u\bar{u}_{xx} - \bar{u}u_{xx} = 2i\operatorname{Im} u\bar{u}_{xx} \\ \iff \frac{d}{dt}(|u|^2) &= 2\operatorname{Im} u\bar{u}_{xx}. \end{aligned} \quad (10)$$

In a similar way we obtain (b).

# Solutions of finite energy

**Lemma 2.-** (Densities conservation). Let  $(u, v)$  be the solution to (1)-(2). Let  $(u_0, v_0) \in L^2(\mathbb{R}) \times L^2(\mathbb{R})$ , then

$$\|u\|_{L^2(\mathbb{R})} = \|u_0\|_{L^2(\mathbb{R})} \quad \text{and} \quad \|v\|_{L^2(\mathbb{R})} = \|v_0\|_{L^2(\mathbb{R})} \quad (11)$$

**Proof.-** Integrating (10) over  $x \in \mathbb{R}$  and using integrating by parts

$$\frac{d}{dt} \|u\|_{L^2(\mathbb{R})}^2 = -2 \operatorname{Im} \int_{\mathbb{R}} |u_x|^2 dx = 0 \iff \frac{d}{dt} \|u\|_{L^2(\mathbb{R})}^2 = 0$$

and integrating over  $t \in [0, T]$  we get the first term. Similarly for  $v$ . The lemma follows.

# Solutions of finite energy

**Lemma 3.-** (Energy conservation). Let  $(u, v)$  be the solution to (1)-(2). Let  $(u_0, v_0) \in L^{2(p+1)}(\mathbb{R}) \times L^{2(p+1)}(\mathbb{R})$ ,  $(u_0, v_0) \in H^1(\mathbb{R}) \times H^1(\mathbb{R})$  and  $p > 1$  odd integer number, then

$$\begin{aligned} & \|u_x\|_{L^2(\mathbb{R})}^2 + \|v_x\|_{L^2(\mathbb{R})}^2 + \frac{1}{p+1} \|u\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} + \frac{1}{p+1} \|v\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} \\ & + \frac{2\beta}{p+1} \int_{\mathbb{R}} |u|^{p+1} |v|^{p+1} dx \\ = & \|u_{0x}\|_{L^2(\mathbb{R})}^2 + \|v_{0x}\|_{L^2(\mathbb{R})}^2 + \frac{1}{p+1} \|u_0\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} + \frac{1}{p+1} \|v_0\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} \\ & + \frac{2\beta}{p+1} \int_{\mathbb{R}} |u_0|^{p+1} |v_0|^{p+1} dx. \end{aligned} \tag{12}$$

# Solutions of finite energy

**Remark.-** From (12), using the Young inequality, we have

$$\begin{aligned} & \|u_x\|_{L^2(\mathbb{R})}^2 + \|v_x\|_{L^2(\mathbb{R})}^2 + \frac{1-\beta}{p+1} \|u\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} + \frac{1-\beta}{p+1} \|v\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} \\ & \leq \|u_{0x}\|_{L^2(\mathbb{R})}^2 + \|v_{0x}\|_{L^2(\mathbb{R})}^2 + \frac{1+\beta}{p+1} \|u_0\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} + \frac{1+\beta}{p+1} \|v_0\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)}. \end{aligned} \quad (13)$$

We follows that, if  $0 < \beta < 1$  then, for all  $1 < p < +\infty$  we have  $u, v \in L^{2(p+1)}(\mathbb{R})$ .

# Estimates for the solutions in $L^\infty(\mathbb{R})$

Throughout this paper job  $C$  is a generic constant, not necessarily the same. at each occasion (it will change from line to line), which depends in an increasing way on the indicated quantities.

We consider the operator  $J$  defined by

$$Ju = e^{ix^2/4t} (2it) \partial_x (e^{-ix^2/4t} u) = (x + 2it\partial_x)u.$$

The  $J$  operator commutes with the operator  $L$  defined by  $L = (i\partial_t + \partial_x^2)$ , this is,  $LJ - JL \equiv [L, J] = 0$ . Moreover,

$$J^m(u) = e^{-ix^2/4t} (2it)^m \partial_x^m (e^{-ix^2/4t} u) = (x + 2it\partial_x)^m u, \quad m \in \mathbb{N},$$

where  $J^m(u) = J(J^{m-1}u)$ ,  $m \in \mathbb{N}$ . Then, applying  $J^m$  to the equations (1)-(2)- we obtain

$$\begin{aligned} i(J^m u)_t + (J^m u)_{xx} &= J^m(|u|^{2p} u) + \beta J^m(|u|^{p-1}|v|^{p+1} u), \quad t > 0, \quad x \in \mathbb{R} \\ i(J^m u)_t + (J^m u)_{xx} &= J^m(|v|^{2p} v) + \beta J^m(|v|^{p-1}|u|^{p+1} v), \quad t > 0, \quad x \in \mathbb{R} \\ J^m u(x, 0) &= x^m u_0(x), \quad J^m u(x, 0) = x^m v_0(x), \quad x \in \mathbb{R}. \end{aligned} \tag{14}$$

We now present our first estimates for the system solutions (1)-(2).

# Estimates for the solutions in $L^\infty(\mathbb{R})$

**Theorem 1.** Let  $0 < \beta < 1$ ,  $p > 1$  odd integer number,

$(u_0, v_0) \in L^{2(p+1)}(\mathbb{R}) \times L^{2(p+1)}(\mathbb{R})$ ,  $(u_0, v_0) \in H^1(\mathbb{R}) \times H^1(\mathbb{R})$  y  
 $(xu_0(x), xv_0(x)) \in L^2(\mathbb{R}) \times L^2(\mathbb{R})$ . Then

$$\|u\|_{L^\infty(\mathbb{R})} \leq \frac{C}{t^{1/4}}, \quad \|v\|_{L^\infty(\mathbb{R})} \leq \frac{C}{t^{1/4}} \quad (15)$$

and

$$\|u\|_{L^{2(p+1)}(\mathbb{R})} \leq \frac{C}{t^{2(p+1)}}, \quad \|v\|_{L^{2(p+1)}(\mathbb{R})} \leq \frac{C}{t^{2(p+1)}}. \quad (16)$$

for all  $t > 0$ .

# Estimates for the solutions in $L^\infty(\mathbb{R})$

**Remark.-** Using the Gagliardo-Nirenberg inequality we have

$$\|u\|_{L^p(\mathbb{R})} \leq C \|u\|_{L^2(\mathbb{R})}^{2/p} \|u\|_{L^\infty(\mathbb{R})}^{(p-2)/p}.$$

Using Lemma 2 and (15) we deduce the following  $L^p$  estimate

$$\|u\|_{L^p(\mathbb{R})} \leq \frac{C}{t^{(p-2)/4p}} \quad \text{and} \quad \|v\|_{L^p(\mathbb{R})} \leq \frac{C}{t^{(p-2)/4p}}.$$

for  $2 < p \leq +\infty$ .

# Some Lemmas

**Lemma 4.-** Let  $k \in \mathbb{N}$ ,  $w \in L^\infty(\mathbb{R})$  and its derivatives of order  $m$ ,  $\partial_x^m w \in L^2(\mathbb{R})$ , then

$$\|\partial_x^m(|w|^{2k} w)\|_{L^2(\mathbb{R})} \leq C_m \|\partial_x^m w\|_{L^2(\mathbb{R})} \|w\|_{L^\infty(\mathbb{R})}^{2k}, \quad (17)$$

where  $C_m$  is a constant that depends of  $m$ .

**Lemma 5.-** Let  $p > 1$  an odd integer number,  $w, z \in L^\infty(\mathbb{R})$  and its derivatives of order  $m$ ,  $\partial_x^m w, \partial_x^m z \in L^2(\mathbb{R})$ , then

$$\|\partial_x^m(|w|^{p-1}|z|^{p+1} w)\|_{L^2(\mathbb{R})} \leq C_m (\|\partial_x^m w\|_{L^2(\mathbb{R})} + \|\partial_x^m z\|_{L^2(\mathbb{R})}) \quad (18)$$

where  $C_m$  is a constant that depends of  $m$ .

# Regularity

**Theorem 2.-** Let  $(u_0, v_0) \in H^1(\mathbb{R}) \times H^1(\mathbb{R})$  with  $(x^n u_0, x^n v_0) \in L^2(\mathbb{R}) \times L^2(\mathbb{R})$  and  $p > 1$  odd integer number. Then there exists a positive constant  $C_m$  depending on  $\|u_0\|_{H^1(\mathbb{R})}$ ,  $\|v_0\|_{H^1(\mathbb{R})}$  and  $\|x^n u_0\|_{L^2(\mathbb{R})}$ ,  $\|x^n v_0\|_{L^2(\mathbb{R})}$  but independent of  $t$  such that

$$\|J^m u\|_{L^2(\mathbb{R})} \leq C_m e^t \quad \text{and} \quad \|J^m v\|_{L^2(\mathbb{R})} \leq C_m e^t, \quad (19)$$

for  $m = 1, 2, \dots, n$ .

# Approximate solution

**Lemma 6.-** Let  $p > 1$  odd integer number, we have  $(u^k, v^k)$  is a Cauchy sequence in  $C([0, T] : H^1(\mathbb{R})) \times C([0, T] : H^1(\mathbb{R}))$  for any  $T > 0$ . Moreover

$$\|u^k - u^j\|_{H^1(\mathbb{R})}^2 + \|v^k - v^j\|_{H^1(\mathbb{R})}^2 \leq C(T) \left[ \|u_0^k - u_0^j\|_{H^1(\mathbb{R})}^2 + \|v_0^k - v_0^j\|_{H^1(\mathbb{R})}^2 \right]$$

where  $C(T)$  is a positive constant independent of  $k$  and  $j$ .

**Lemma 7.-** Let  $p > 1$  odd integer number, for  $m = 1, 2, 3, \dots, n$ , we have

$\{J^m u^k, J^m v^k\}$  is Cauchy sequence in  $C([0, T] : L^2(\mathbb{R})) \times C([0, T] : L^2(\mathbb{R}))$  for any  $T > 0$ . Moreover

$$\begin{aligned} & \|J^m u^k - J^m u^j\|_{L^2(\mathbb{R})}^2 + \|J^m v^k - J^m v^j\|_{L^2(\mathbb{R})}^2 \\ & \leq c(T) \left[ \|x^m u_0^k - x^m u_0^j\|_{L^2(\mathbb{R})}^2 + \|x^m v_0^k - x^m v_0^j\|_{L^2(\mathbb{R})}^2 \right] \end{aligned}$$

where  $c(T)$  is a positive constant independent of  $k$  and  $j$ .

# Main Theorem

## Theorem.-

- Let  $p > 1$  odd integer number,  $(u_0, v_0) \in H^1(\mathbb{R}) \times H^1(\mathbb{R})$  and  $(x^n u_0, x^n v_0) \in L^2(\mathbb{R}) \times L^2(\mathbb{R})$  for some  $n \in \mathbb{N}$ . Then, there exists a unique solution  $(u(x, t), v(x, t))$  of (1)-(2) satisfying

$$(u, v) \in C_b(\mathbb{R} : H^1(\mathbb{R})) \times C_b(\mathbb{R} : H^1(\mathbb{R})) \quad (20)$$

$$(J^m u, J^m v) \in C(\mathbb{R} : L^2(\mathbb{R})) \times C(\mathbb{R} : L^2(\mathbb{R})), \quad m = 1, 2, \dots, n. \quad (21)$$

Moreover  $(u, v)$  satisfies the integral identities:

Densities Conservation

$$\|u\|_{L^2(\mathbb{R})} = \|u_0\|_{L^2(\mathbb{R})} \quad \text{and} \quad \|v\|_{L^2(\mathbb{R})} = \|v_0\|_{L^2(\mathbb{R})}.$$

Energy Conservation

$$\begin{aligned} & \|u_x\|_{L^2(\mathbb{R})}^2 + \|v_x\|_{L^2(\mathbb{R})}^2 + \frac{1}{p+1} \|u\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} + \frac{1}{p+1} \|v\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} \\ & + \frac{2\beta}{p+1} \int_{\mathbb{R}} |u|^{p+1} |v|^{p+1} dx \\ &= \|u_{0x}\|_{L^2(\mathbb{R})}^2 + \|v_{0x}\|_{L^2(\mathbb{R})}^2 + \frac{1}{p+1} \|u_0\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} + \frac{1}{p+1} \|v_0\|_{L^{2(p+1)}(\mathbb{R})}^{2(p+1)} \\ & + \frac{2\beta}{p+1} \int_{\mathbb{R}} |u_0|^{p+1} |v_0|^{p+1} dx. \end{aligned}$$

# Main Theorem

2. Let  $(u_0, v_0) \in H^1(\mathbb{R}) \times H^1(\mathbb{R})$  with  $(x^n u_0, x^n v_0) \in L^2(\mathbb{R}) \times L^2(\mathbb{R})$  and  $p > 1$  odd integer number. Then there exists a positive constant  $C_m$  depending on  $\|u_0\|_{H^1(\mathbb{R})}$ ,  $\|v_0\|_{H^1(\mathbb{R})}$  and  $\|x^n u_0\|_{L^2(\mathbb{R})}$ ,  $\|x^n v_0\|_{L^2(\mathbb{R})}$  but independent of  $t$  such that

$$\|J^m u\|_{L^2(\mathbb{R})} \leq C_m e^t \quad \text{and} \quad \|J^m v\|_{L^2(\mathbb{R})} \leq C_m e^t, \quad (22)$$

for  $m = 1, 2, \dots, n$ .

3. Let  $\beta < 1$ ,  $p > 1$  odd integer number,  $(u_0, v_0) \in L^{2(p+1)}(\mathbb{R}) \times L^{2(p+1)}(\mathbb{R})$ ,  $(u_0, v_0) \in H^1(\mathbb{R}) \times H^1(\mathbb{R})$  and  $(xu_0(x), xv_0(x)) \in L^2(\mathbb{R}) \times L^2(\mathbb{R})$ . Then, for all  $t \neq 0$  we have

$$\|u\|_{L^\infty(\mathbb{R})} \leq \frac{C}{t^{1/4}} \quad \text{and} \quad \|v\|_{L^\infty(\mathbb{R})} \leq \frac{C}{t^{1/4}}$$

and

$$\|u\|_{L^{2(p+1)}(\mathbb{R})} \leq \frac{C}{t^{2(p+1)}} \quad \text{and} \quad \|v\|_{L^{2(p+1)}(\mathbb{R})} \leq \frac{C}{t^{2(p+1)}}.$$

# Main Theorem

**Proof of the main theorem.-** From Lemma 6 and 7 we obtain that there exists  $u = u(x, t)$  and  $v = v(x, t)$  satisfying (20)-(21) and such that for any  $T > 0$  we have

$$\begin{aligned} u^k &\longrightarrow u \text{ strongly in } C(\mathbb{R} : H^1(\mathbb{R})) \\ v^k &\longrightarrow v \text{ strongly in } C(\mathbb{R} : H^1(\mathbb{R})) \end{aligned}$$

and

$$\begin{aligned} J^m u^k &\longrightarrow J^m u \text{ strongly in } C(\mathbb{R} : L^2(\mathbb{R})) \\ J^m v^k &\longrightarrow J^m v \text{ strongly in } C(\mathbb{R} : L^2(\mathbb{R})). \end{aligned}$$

It is easily verified that  $(u, v)$  solves (1)-(2) and satisfies (11)-(12).

# Remark and corolary

**Remark.-** If the assumption (22) holds, then

$$e^{\frac{ix^2}{4t}} u \in C(\mathbb{R} - \{0\} : H^m(\mathbb{R})), \quad e^{\frac{ix^2}{4t}} v \in C(\mathbb{R} - \{0\} : H^m(\mathbb{R})).$$

**Corollary.-** If  $(x^n u_0, x^n v_0) \in L^2(\mathbb{R}) \times L^2(\mathbb{R})$  for all  $n \in \mathbb{N}$ , then the solution  $(u, v)$  of (1)-(2) is infinitely differentiable in  $x$  and  $t$  for  $t \neq 0$ .

# Bibliografía

1. G.P. Agarwal. *Nonlinear Fiber Optics*. 2nd edition. Academic Press, New York. 1995.
2. F.W. Sears, M.W. Zemansky, H.D. Young, R.A. Freedman. *Física Universitaria*. Vol 2. Impreso en México. 2004.
3. R.A. Serway, J.W Jewett Jr. *Física para ciencias e ingenieria*. Vol 2, Novena edición. Impreso en Mexico. 2015.
4. M. Alves, M. Sepúlveda, O. Vera. *Smoothing properties for the higher-order nonlinear Schrödinger equation with constant coefficients*. J. Nonlinear Analysis. Vol. 71, pp. 948-966, 2009.
5. B. Alouini. *A note on the finite fractal dimension of the global attractors for dissipative nonlinear Schrödinger-type equations*. Math Meth Appl Sci. Vol. 44, pp. 91-103, 2021.

# Bibliografía

6. V. Bisognin, C. Buriol, M. Sepúlveda, O. Vera, *Asymptotic Behaviour for a Nonlinear Schrödinger Equation in Domains with Moving Boundaries*. Acta Appl Math. Vol. 125, pp. 159-172, 2013.
7. H. Brézis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*. Springer, New York, 2010.
8. J. C. Ceballos, M. Sepúlveda, O. Vera. *Gain in regularity for a coupled nonlinear Schrödinger system*. Bol. Soc. Param. Mat. Vol. 24, pp. 1-4, 2006.
9. X. Carvajal, P. Gamboa, S. Necasova, H. H. Nguyen, O. Vera. *Asymptotic behavior of solutions to a system of Schrödinger equations*. Electronic Journal of Differential Equations. Vol. 2017, pp. 1-23, 2017.
10. T. Cazenave, Z. Han, Y. Martel. *Blowup on an Arbitrary Compact Set for a Schrödinger Equation with Nonlinear Source Term*. J. Dynamics and Differential Equations. Vol. 33, pp. 941-960, 2021.

# Bibliografía

11. T. Cazenave, F. Weissler, *The Cauchy problem for the critical nonlinear Schrödinger equation en  $H^s$* . Nonlinear Analysis, Theory, Methods and Applications. Vol. 14, No. 10, pp. 807-836, 1990.
12. T. Cazenave, Fred B. Weissler, *Rapidly Decaying Solutions of the Nonlinear Schrödinger Equation*. Commun. Math. Phys. Vol. 147, pp. 75-100, 1992.
13. T. Cazenave, Z. Han, I. Maumkin. *Asymptotic behavior for a dissipative nonlinear Schrödinger equation*. Nonlinear. Analysis. Vol. 205, pp. 112243, 2021.
14. T. Cazenave, F. Dickstein, Fred B. Weissler. *Non-regularity in Hölder and Sobolev spaces of solutions to the semilinear heat and Schrödinger equations*. Nagoya Math. J. Vol. 226, pp. 44-70, 2017.
15. Marcelo M. Cavalcanti, Wellington J. Correa, Andrei V. Faminskii, Mauricio A. Sepúlveda C., R. Véjar-Asem. *Well-posedness and asymptotic behavior of a generalized higher order nonlinear Schrödinger equation with localized dissipation*. Computers and Mathematics with Applications. Vol. 96, pp. 188-208, 2021.

# Bibliografía

16. M. M. Cavalcanti, V. N. Cavalcanti, A. Guesmia, M. Sepúlveda. *Well-Posedness and Stability for Schrödinger Equations with Infinite Memory.* Applied Mathematics and Optimization. Vol. 85, pp. 20, 2022.
17. N. Hayashi, Pavel I. Naumkin, *Scattering Problem for the Supercritical Nonlinear Schrödinger Equation in 1d.* Funkcialaj Ekvacioj. Vol. 58, pp. 451-470, 2015.
18. N. Hayashi, Pavel I. Naumkin. *Domain and Range of the Modified Wave Operator for Schrödinger Equations with a Critical Nonlinearity.* Commun. Math. Phys. Vol. 267, pp. 477-492, 2006.
19. N. Hayashi, K. Nakamitsu, M. Tsutsumi. *On Solutions of the Initial Value Problem for the Nonlinear Schrödinger Equations.* Journal of Functional Analisys. Vol. 71, pp. 218-245, 1987.
20. N. Hayashi, K. Nakamitsu, M. Tsutsumi. *On solutions of the initial value problem for the nonlinear Schrödinger equations in one space dimension.* Math. Z. 192. Vol. 4, pp. 637-650, 1986.

# Bibliografía

21. Y. Liao, Q. Sun, X. Zhao, M. Cheng. *Asymptotic stability of standing waves for the coupled nonlinear Schrödinger system.* Liao et al. Boundary Value Problem. Vol. 3, 2015.
22. L. Ma, X. Song, L. Zhao. *On global rough solutions to a non-linear Schödinger system.* Glasgow Math. J. Vol. 51, pp. 499-511, 2009.
23. L. Ma, L. Zhao. *Uniqueness of ground states of some coupled nonlinear Schrödinger systems and their application.* J. Differential Equations. Vol. 245, pp. 2551-2565, 2008.
24. L. Ma, L. Zhao. *On energy stability for the coupled nonlinear Schrödinger system.* Z. angew. Math. Phys. Vol. 60, pp. 744-784, 2009.
25. S. Min. *Existence and uniqueness of global solutions to the initial-boundary value problem for coupled nonlinear Schrödinger systems.* J. Xinjiang Univ. Nat. Sci. (9) Vol. 2, pp. 29-34, 1992.

# Bibliografía

26. J. Muñoz Rivera, V. Poblete, M. Sepúlveda, H. Vargas, O. Vera. *Remark on the stabilization for a Schrödinger equation with double power nonlinearity.* Applied Mathematics Letters. Vol. 98, pp. 63-69, 2019.
27. D. C. Roberts, A. C. Newell. *Finite-time collapse of N classical fields described by coupled nonlinear Schrödinger equations.* Phys. Rev. Vol. 3, pp. 74, 2006.
28. K. Wang, D. Zhao, B. Feng. *Optimal bilinear control of the coupled nonlinear Schrödinger system.* Nonlinear Analysis: Real World Applications. Vol. 47, pp. 142-167, 2019.

# Existence and Uniqueness

Thank you