THE EMBEDDING PROBLEM FOR HOMEOMORPHISMS VERSUS THE STABILITY OF ROTATION SETS ON $\mathbb{T}^d$.

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Abstract

We describe topological obstructions (involving periodic points, topological entropy and rotation sets) for a homeomorphism on a compact manifold to embed in a continuous flow.

We prove that homeomorphisms in a C^0-open and dense set of homeomorphisms isotopic to the identity in compact manifolds of dimension at least two are not the time-1 map of a continuous flow. Such property is also true for volume preserving homeomorphisms in compact manifolds of dimension at least five.

In the case of conservative homeomorphisms of the torus $\mathbb{T}^d$ ($d \geq 2$) isotopic to identity, we also proved that there exists a C^0-open and dense with a stable rotation set, for every $d \geq 3$. In addition, the rotation set of every such homeomorphism is a polyhedron with rational vertices and non-empty interior. In this case, we describe necessary conditions for a homeomorphism to be flowable in terms of the rotation sets.

References

[1] J. Franks, Recurrence and fixed points of surface homeomorphisms. Ergodic Theory Dynam. Systems 8 (1988), Charles Conley Memorial Issue, 99-107.

[2] P.-A. Guiheneuf, A. Koropecki, Stability of the rotation set of area-preserving toral homeomorphisms, *Nonlinearity* 30 (2017) 1089–1096.

[3] J. Kwapisz, Every convex polygon with rational vertices is a rotation set. *Ergod. Th. Dyn. Syst.* 12 (1992) 333–339.

[4] H. Lima and P. Varandas, On the rotation sets of generic homeomorphisms on the torus $\mathbb{T}^d$, Preprint ArXiv:1901.00396

[5] J. Llibre, R. MacKay, Rotation vectors and entropy for homeomorphisms of the torus homotopic to the identity. *Ergod. Th. Dyn. Syst.* 11 (1991), no. 1, 115-128.

[6] M. Misiurewicz, K. Ziemian, Rotation sets and ergodic measures for torus homeomorphisms. *Fund. Math.* 137 (1991), no. 1, 45-52.

[7] M. Misiurewicz, *Rotation Theory*, Online Proceedings of the RIMS Workshop "Dynamical Systems and Applications: Recent Progress" (2006).

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[8] A. Passeggi, Rational polygons as rotation sets of generic homeomorphisms of the two torus. *J. Lond. Math. Soc.* (2) 89 (2014), no. 1, 235-254.

[9] L. Polterovich and E. Shelukhin, Autonomous Hamiltonian flows, Hofer's geometry and persistence modules, *Sel. Math.* New Series 22 (2016) 227–296.

[10] C. Pugh and C. Robinson, The C^1-Closing Lemma, including Hamiltonians, *Ergodic Th. Dynam. Sys.* 3, (1983) 261–313.

[11] J.-C. Sikorav, Approximation of a volume-preserving homeomorphism by a volume-preserving diffeomorphism, http://www.umpa.ens-lyon.fr/ symplexe, September 2007.

[12] P. Teixeira On the conservative pasting lemma, *Ergodic Th. Dynam. Sys.* (2018) https://doi.org/10.1017/etds.2018.81

[13] W. Utz, The embedding of homeomorphisms in continuous flows, *The Proceedings of the 1981 Topology Conference (Blacksburg, Va., 1981). Topology Proc.* 6 (1982), no. 1, 159–177.

[14] M. Zdun, *On embedding of homeomorphisms of the circle in a continuous flow. Iteration theory and its functional equations* (Lochau, 1984), 218–231, Lecture Notes in Math., 1163, Springer, Berlin, 1985.

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