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Deductibility, almost topology and operators on spaces of deductions

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Abstract

The concept of 'almost topological space' is a generalization of the usual notion of topological space by the exclusion of some axioms, such that every topological space is an almost topological space, but do not hold the reverse.

Definition: An *almost topological space* is a pair (E, Ω) such that E is a non-empty set and $\Omega \subseteq \mathcal{P}(E)$ satisfies the following condition:

$$B \subseteq \Omega \Rightarrow \cup B \in \Omega.$$

Definition: The collection Ω is called *almost topology* and each member of Ω is an *open* of (E, Ω). A set $A \in \mathcal{P}(E)$ is *closed* when its complement relative to E is an open of (E, Ω).

Its motivation comes from the definition of Tarski consequence operator.

Tarski introduced his definition of consequence to explicit the most important notions that any logical consequence must share.

Definition: A *consequence operator* on E is a function $^- : \mathcal{P}(E) \to \mathcal{P}(E)$ such that, for every $A, B \subseteq E$:
(i) $A \subseteq \overline{A}$;
(ii) $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$;
(iii) $\overline{\overline{A}} \subseteq \overline{A}$.

Of course, from (i) and (iii), the equality $\overline{\overline{A}} = \overline{A}$ holds, for every $A \subseteq E$.

Definition: A consequence operator $^- : \mathcal{P}(E) \to \mathcal{P}(E)$ is *finitary* when, for every $A \subseteq E$:

$$\overline{A} = \cup\{\overline{A}_f : A_f \text{ is a finite subset of } A\}.$$

Definition: A *Tarski' space* (*Tarski's deductive system* or *Closure space*) is a pair $(E, ^-)$ such that E is a non-empty set and $^-$ is a consequence operator on E.

The the concepts of Tarski' space and almost topology are inter-definable.

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We present an algebraic structure relative to Tarski's spaces.

Definition: A *TK-algebra* is a sextuple $\mathcal{A} = (A, 0, 1, \vee, \sim, \bullet)$ such that $(A, 0, 1, \vee, \sim)$ is a Boolean algebra and $\bullet$ is a new operator, called *operator of Tarski*, such that:
(i) $a \vee \bullet a = \bullet a$;
(ii) $\bullet a \vee \bullet(a \vee b) = \bullet(a \vee b)$;
(iii) $\bullet(\bullet a) = \bullet a$.

From the TK-algebras it was introduced the Logic **TK**, that has as model exactly the closed sets in any Tarski's space.

The propositional logic **TK** is constructed over the propositional language $L = \{\neg, \vee, \rightarrow, \blacklozenge, p_1, p_2, p_3, ...\}$ with the following axioms and rules:

(CPC) φ, if φ is a tautology

(TK_1) $\varphi \rightarrow \blacklozenge\varphi$

(TK_2) $\blacklozenge\blacklozenge\varphi \rightarrow \blacklozenge\varphi$

(MP) $\dfrac{\varphi \rightarrow \psi, \varphi}{\psi}$

$(RM^{\blacklozenge})$ $\dfrac{\vdash \varphi \rightarrow \psi}{\vdash \blacklozenge\varphi \rightarrow \blacklozenge\psi}.$

The TK-algebras are algebraic models for the logic **TK**.

In this paper we have proposed to investigate operators over an almost topology (E, Ω) and its relation with usual modal operators, in the tradition of Kripke.

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