

The Mandelbrot set and its Satellite copies

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Abstract

For a polynomial on the Riemann sphere, infinity is a (super) attracting fixed point, and the filled Julia set is the set of points with bounded orbit. Consider the quadratic family $P_c(z) = z^2 + c$. The Mandelbrot set M is the set of parameters c such that the filled Julia set of P_c is connected.

Computer experiments quickly reveal the existence of small homeomorphic copies of M inside itself; the existence of such copies was proved by Douady and Hubbard. Each little copy is either *primitive* (with a cusp on the boundary of its main cardioid region) or a *satellite* (without a cusp). Lyubich proved that the primitive copies of M satisfy a stronger regularity condition: they are *quasiconformally* homeomorphic to M . The satellite copies are not quasiconformally homeomorphic to M (as we cannot straighten a cusp quasiconformally), but are they mutually quasiconformally homeomorphic? In joint work with C. Petersen we prove that the answer is negative in general, but positive in the case the satellite copies have rotation numbers with the same denominator.

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