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Berry-Esseen estimates based on generators with applications.

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Abstract

The theme of functional central limit theorems (functional CLTs) comes from the now standard Donsker Theorem and Invariance Principles for the Brownian motion. Since then, a large bibliography has been developed on the invariance principles of many kinds of random walks (on random media for instance) and its convergence to standard Brownian motions. On the other hand, convergence to Brownian-type process like the skew Brownian motion, the stick Brownian motion, the elastic Brownian motion and many others is surprisingly much less studied up to the current days. As an example, in [**?**] it was presented a functional central limit for the slow bond random walk, which limit is given by the snapping out Brownian motion (abreviatted by SNOB), a Brownian type process created only in 2016 by [**?**].

In this work, based on convergence of generators, we present a broad criteria to assure a functional central limit. This criteria holds, in principle, to any limit Feller process, not only a Brownian motion. Such kind of result based on generators is not new: it comes from the sixties by many authors. See Ethier and Kurtz book [**?**, Theorem 6.1 page 28] and references therein in pages 47–48.

Our result is less general than [**?**, Theorem 6.1 page 28] in sense that result is an equivalence. On the other hand, our result gives information about the speed of convergence, that is, Berry-Esseen estimates (for finite-dimensional distributions). Since the convergence in distribution can be assured via convergence under a plenty of different metric (the Lévy metric, the Prohorov metric, the dual Lipzchtz metric, etc.), the Berry-Essen estimates for a given CLT can be formulated in many ways as well. In our result, the Berry-Essen estimates we provide are formulated in terms of a metric coming naturally from the generator of the limiting process. Though it is not a standard way of stating a Berry-Essen estimates, it has the advantage of working for all applications at same time, sparing the efforts of analyzing each individual problem.

Of course, the speed of convergence of a discrete process towards a continuous Feller process may occur at arbitrary slow speed of convergence: our main result is not a panacea at all. However, it covers many interesting and new cases of functional central limit theorems for classical Brownian-type process which, to the best of our knowledge, had been considered in the literature. In other words, having a given Feller process as a target, suitably choosing the (usuallly discrete) model we will be able to provide a

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corresponding functional CLT with Berry-Essen estimates, what is also useful in the context of simulations of stochastic processes and applications.

References

Apresentação oral: